SU(3) Sum Rules in Charm Decay

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Based on: 1211.3361 (Y. Grossman and DR);
1203.6659 (J. Brod, A. Kagan, Y. Grossman, and J. Zupan).

Some Motivation: Direct $\not{CP}$ in $D \to KK, \pi\pi$

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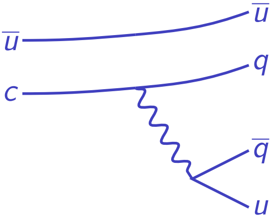

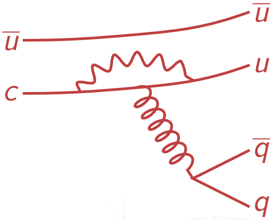

In U-spin limit, $q = (d, s) \sim 2$ of $\mathrm{SU}(2)_U \implies$

$\simeq V_{cq} V^*_{uq} T = \pm\lambda T$

$\simeq -V_{cb} V^*_{ub} P = \lambda^5(\rho - i\eta)P$

Background: Direct $\Delta\mathcal{A}_{\rm CP}$

Direct CP asymmetry for CP definite final states

$$\mathcal{A}_{\rm CP}(D \to f) = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to f)}{\Gamma(D \to f) + \Gamma(\overline{D} \to f)}\,.$$

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$$P/T \sim \frac{\alpha_S(m_c)}{\pi} \sim 0.15\,, \quad \text{or} \sim \frac{\alpha_S(m_c)}{\alpha_S(m_b)} \frac{P}{T}\bigg|_{\mathrm{B}} \implies \boxed{P/T \sim 0.3}\,.$$

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Is this new physics? Or SM?

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Today's Story

- A consistent picture of $\Delta\mathcal{A}_{\mathrm{CP}}$ and other data can be obtained within SM via approximate flavor SU(3) and QCD non-perturbative effects. Need enhancement of $\cancel{\mathrm{SU}(3)}$ reduced matrix elements. Can globally fit subset to larger space of data. E.g. $\Delta U = 0$ rule, and other schemes.

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- Patterns of $\cancel{\mathrm{SU(3)}}$ imply sum rules. What are they for the "usual pattern" of $\cancel{\mathrm{SU(3)}}$ beyond leading order?
- Certain sum rules provide very sensitive tests of $\cancel{\mathrm{SU(3)}}$ (and hence possibly NP) and predictions.

Plan

- Some Motivation
- **Warm-up: U spin and $\Delta U = 0$ rule**
- Flavor SU(3) picture
  - Formalism
  - Isospin Probes
  - Rate sum rules
  - Predictions

Historic SU(3) data

Naïve $\cancel{SU(3)}$ parameter:

$$\boxed{\varepsilon \equiv f_K/f_\pi - 1 \sim 0.2}$$

NB: Defined at amplitude level.

Historic SU(3) data

- Large $\cancel{SU(3)}$:

$$\left|\frac{\mathcal{A}_{D^0\to K^+K^-}}{\mathcal{A}_{D^0\to\pi^+\pi^-}}\right| - 1 \simeq 0.8 \qquad \sim \mathcal{O}(1)$$

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What's going on? Is this related to $\Delta\mathcal{A}_{\mathrm{CP}}$?

Detour: $\Delta I = 1/2$ Rule

A similar problem arises in the $K \to \pi\pi$ system: isospin 2 amplitudes are suppressed compared to isospin 0. Embed the pions and kaons into isospin 1 and 1/2 irreps

$$\Pi = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}~, \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}~.$$

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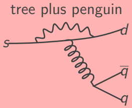
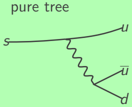
$K^*\Pi\Pi$ must form a singlet with the Hamiltonian, so roughly

$$H = X_{1/2}\, H_{1/2} + X_{3/2}\, H_{3/2}$$





$$A(K_S \to 2\pi^0) = \sqrt{\frac{2}{3}}X_{1/2} - \frac{2}{\sqrt{3}}X_{3/2}$$

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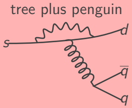

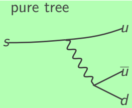

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Rule: Penguins are enhanced by QCD final state effects $X_{1/2}/X_{3/2} \sim 22$.

$\Delta U = 0$ Rule

- Embed final states into U-spin irreps

$$\begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} \sim 2 \qquad \begin{pmatrix} \pi^- \\ K^- \end{pmatrix} \sim 2$$

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$$H = X_0 H_0 + X_1 H_1$$

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- Extra ingredient: U-spin broken by strange quark mass spurion

$$m_s = \varepsilon \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \sim 3$$

- Spurion produces a power series expansion order by order in m_s. I.e.

$$H = X_0 H_0 + X_1 H_1 + X_0^{(1)}(Hm_s)_0 + X_1^{(1)}(Hm_s)_1 + \ldots$$

$\Delta U = 0$ Rule

Compute amplitudes in terms of reduced matrix elements to first order in U-spin breaking by m_s.

$$\begin{aligned}
\mathcal{A}_{D^0\to K^+\pi^-} &= \lambda^2(T_0 + \varepsilon T_1) \\
\mathcal{A}_{D^0\to K^-\pi^+} &= \quad (T_0 - \varepsilon T_1) \\
\mathcal{A}_{D^0\to K^+K^-} &= +\lambda(T_0 - \varepsilon \boxed{P_1}) + \lambda^5 \boxed{P_0} \\
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- $\Delta U = 0$ penguins P_0 and P_1 are enhanced by factor ~ 5.
- This is a non-perturbative QCD effect: 'penguin enhancement'.

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Question: Can we extend this to full SU(3) picture? If we do, we expect more sum rules.

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Formalism

Interested in relations involving amplitudes

$$A_{\mu\to\alpha\beta} \equiv \left\langle M_\alpha N_\beta \middle| H \middle| D_\mu \right\rangle \,.$$

M_α, N_β and D_μ are pseudoscalar or vector meson states embedded into SU(3) irreps. $q \sim (u, d, s)$ and

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$$D = \begin{pmatrix} D^0 \\ D^+ \\ D_s^+ \end{pmatrix}\,, \qquad P_1 = \eta_1\,, \qquad V_1 = \phi_1\,,$$

$$P_8 = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix}\,,$$

$$V_8 = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho_0 + \frac{1}{\sqrt{6}}\omega_8 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho_0 + \frac{1}{\sqrt{6}}\omega_8 & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\sqrt{\frac{2}{3}}\omega_8 \end{pmatrix}\,.$$

Formalism

Wigner Eckart Theorem

$$\text{Amplitudes} = \sum \text{RMEs} \times \text{Clebsch-Gordan}$$

Formalism

Reduced Matrix Element

SU(3) Invariant/ Clebsch-Gordan

Wigner-Eckart Th'm

$$A_{\mu\to\alpha\beta} = \sum_w X_w \, (C_w)_{\alpha\beta\mu}\,,$$

$$(C_w)_{\alpha\beta\mu} = \frac{\partial^3}{\partial M_\alpha \partial N_\beta \partial D_\mu}\Big[M^i_j N^k_l H^{p_1\cdots p_n}_{q_1\cdots q_m} D^r\Big]_w\,.$$

NB: by completeness $H = \sum_w X_w C_w$.

4-Quark Hamiltonian

Here 4-quark Hamiltonian, with $q \sim (u, d, s)$

$$H_0 = \bar{q}q\bar{q}c \sim \bar{3}_p \oplus \bar{3}_t \oplus 6 \oplus \overline{15}$$

H_0 breaks SU(3)!

Penguin

CF, SCS and DCS

Under C-G decomposition, obtain non-zero components, e.g.

$$[\overline{\mathbf{15}}]^3_{12} = \frac{1}{2}[(\bar{u}s)(\bar{d}c) + (\bar{d}s)(\bar{u}c)] \mapsto V_{us}V^*_{cd}/2 = -\lambda^2/2$$

SU(3) Breaking

Dominant breaking from s-quark mass

$$m_s = \varepsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}~, \quad \varepsilon \sim 0.2~.$$

Also isospin breaking,

$$m_I = \delta \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}~, \quad \delta = (m_u - m_d)/\Lambda_{\mathrm{qcd}} \sim 1\%~,$$

Hamiltonian

$$H = H_0 + H_0 m_s + H_0 m_I + \ldots~.$$

Note: $m_s : SU(3) \to SU(2)_I \times U(1)_{\mathrm{EM}}$ doesn't (further) break isospin

Sum Rules

Wigner-Eckart:

$$\begin{pmatrix} A_1 \\ \vdots \\ A_n \end{pmatrix} = \begin{pmatrix} C_{11} & \dots & \\ \vdots & \ddots & \\ & & C_{nw} \end{pmatrix} \begin{pmatrix} X_1 \\ \vdots \\ X_w \end{pmatrix}$$

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A sum rule is a symbol $\mathcal{S}$:

$$\boxed{\mathcal{S}^{\alpha\beta\mu}\mathcal{A}_{\alpha\beta\mu} = 0} \quad \Leftrightarrow \quad \boxed{\mathcal{S}^{\alpha\beta\mu}C_{w\,\alpha\beta\mu} = 0\ , \quad \forall w}\ .$$

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$\mathcal{O}(\epsilon^p)$ sum rule algorithm:

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Many $\mathcal{O}(\varepsilon)$ sum rules: 14 for PP and 27 for PV! (see 1211.3361 for a full list)

$\mathcal{O}(\varepsilon^2)$ Sum Rule Examples

U-spin rules:

$$\frac{\mathcal{A}_{D^0\to K^-K^+}}{\lambda} + \frac{\mathcal{A}_{D^0\to \pi^-K^+}}{\lambda^2} - \mathcal{A}_{D^0\to K^-\pi^+} - \frac{\mathcal{A}_{D^0\to \pi^-\pi^+}}{\lambda} = 0$$

$$-\mathcal{A}_{D^0\to K^{*-}\pi^+} + \frac{\mathcal{A}_{D^0\to K^{*-}K^+}}{\lambda} - \frac{\mathcal{A}_{D^0\to \rho^-\pi^+}}{\lambda} + \frac{\mathcal{A}_{D^0\to \rho^-K^+}}{\lambda^2} = 0$$

$$\mathcal{A}_{D^0\to K^-\rho^+} + \frac{\mathcal{A}_{D^0\to \pi^-\rho^+}}{\lambda} - \frac{\mathcal{A}_{D^0\to K^-K^{*+}}}{\lambda} - \frac{\mathcal{A}_{D^0\to \pi^-K^{*+}}}{\lambda^2} = 0$$

$\mathcal{O}(\varepsilon^2)$ Sum Rule Examples

Other examples:

$$-\frac{\sqrt{3}\mathcal{A}_{D^0\to\eta_8K^0}}{\lambda^2} + \frac{\mathcal{A}_{D^0\to K^0\pi_0}}{\lambda^2} - \sqrt{3}\mathcal{A}_{D^0\to\eta_8\overline{K}^0} + \mathcal{A}_{D^0\to\pi_0\overline{K}^0} = 0$$

$$\frac{\sqrt{2}\mathcal{A}_{D^+\to\pi_0K^+}}{\lambda^2} - \frac{\mathcal{A}_{D^+\to\overline{K}^0K^+}}{\lambda} - \frac{\sqrt{2}\mathcal{A}_{D^+\to\pi_0\pi^+}}{\lambda} + \mathcal{A}_{D^+\to\overline{K}^0\pi^+}$$
$$+\frac{\sqrt{2}\mathcal{A}_{D_s^+\to\pi_0K^+}}{\lambda} - \mathcal{A}_{D_s^+\to\overline{K}^0K^+} = 0$$

$$-\frac{\mathcal{A}_{D^0\to\eta_8\omega_8}}{\lambda} + \sqrt{\frac{3}{2}}\frac{\mathcal{A}_{D^0\to\omega_8K^0}}{\lambda^2} + \frac{\mathcal{A}_{D^0\to\eta_8\rho_0}}{\sqrt{3}\lambda} - \frac{\mathcal{A}_{D^0\to K^0\rho_0}}{\sqrt{2}\lambda^2} + \sqrt{\frac{2}{3}}\frac{\mathcal{A}_{D^0\to\eta_8K^{*0}}}{\lambda^2}$$
$$-\sqrt{\frac{2}{3}}\mathcal{A}_{D^0\to\eta_8\overline{K}^{*0}} + \frac{\mathcal{A}_{D^0\to K^0\overline{K}^{*0}}}{\lambda} = 0$$

Mixing

$$\begin{pmatrix} K_S \\ K_L \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

$$\begin{pmatrix} \omega \\ \phi \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix} \begin{pmatrix} \omega_8 \\ \phi_1 \end{pmatrix}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}$$

Don't consider $K^* - \overline{K}^*$ mixing: flavor states can be tagged.

Abstract Generation

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- E.g. H_0 is invariant under $S \equiv -\lambda H^U - \lambda^2 E^U_- + E^U_+$. A sum rule for each (QED-appropriate) choice of $\alpha\beta\mu$!

$$0 = S^{\rho\sigma\gamma}_{\pi^+\pi^-D^0}[C_w]_{\rho\sigma\gamma}/\lambda^2 = -\frac{[C_w]_{\pi^-K^+D^0}}{\lambda^2} - [C_w]_{K^-\pi^+D^0}\,.$$

Plan

- Some Motivation
- Warm-up: U spin and $\Delta U = 0$ rule
- Flavor SU(3) picture
  - Formalism
  - Isospin Probes
  - Rate sum rules
  - Predictions

Isospin Sum Rules

- Isospin sum rules hold to all orders of $\cancel{\mathrm{SU}(3)}$ by m_s, i.e. to all orders in ε.
- $\implies$ Isospin sum rules valid at $\mathcal{O}(\epsilon^0, \delta)$ are therefore expected to hold to $\delta^2 \sim 10^{-4}$. Very sensitive test of SM $\cancel{\mathrm{SU}(3)}$ pattern.

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- Convenient way to see isospin sum rules are unbroken by m_s to all orders. Proof: If tensor T_I generates sum rules under embedding into isospin subgroup ("isospin sum rules"), then $T_I m_s^p = 0$. Hence $T_I H_0(1 + m_s + ...) = 0$.

Isospin Sum Rules

- $\mathcal{O}(\epsilon^0, \delta)$ for PP: (only one)

$$\frac{\lambda\mathcal{A}_{D^0\to\pi^-\pi^+} - \lambda\sqrt{2}\mathcal{A}_{D^0\to 2\pi_0} + \lambda\sqrt{2}\mathcal{A}_{D^+\to\pi_0\pi^+}}{\sqrt{2}\mathcal{A}_{D^0\to K^0\pi_0} + \mathcal{A}_{D^0\to\pi^-K^+} - \mathcal{A}_{D^+\to K^0\pi^+} + \sqrt{2}\mathcal{A}_{D^+\to\pi_0 K^+}} - 1$$

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- $\mathcal{O}(\epsilon^0, \delta^2)$ for PV: (examples)

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$$\mathcal{A}_{D^0\to K^0\rho_0} + \frac{\mathcal{A}_{D^0\to\rho^-K^+}}{\sqrt{2}} + \mathcal{A}_{D^+\to\rho_0 K^+} - \frac{\mathcal{A}_{D^+\to K^0\rho^+}}{\sqrt{2}} = 0$$

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- NB: Need strong phases to measure these. Can be difficult to obtain.

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Rate Sum Rules

Wigner-Eckart for square amplitudes

$$\left|\mathcal{A}_{\mu\to\alpha\beta}\right|^2 = \sum_{w}\sum_{w'} X_w X^*_{w'} (C_w)_{\alpha\beta\mu} (C_{w'})_{\alpha\beta\mu}$$
$$\equiv \sum_u X_u (C_u)_{\alpha\beta\mu}$$

Finding C_u kernel determines rate sum rules. No strong phases!

$\mathcal{O}(\epsilon^2)$ Rate Sum Rule Examples

U-spin rules

$$\frac{|\mathcal{A}_{D^0\to K^-K^+}|^2}{\lambda^2} + \frac{|\mathcal{A}_{D^0\to \pi^-\pi^+}|^2}{\lambda^2} = \frac{|\mathcal{A}_{D^0\to \pi^-K^+}|^2}{\lambda^4} + |\mathcal{A}_{D^0\to K^-\pi^+}|^2$$

$$\frac{|\mathcal{A}_{D^0\to \rho^-K^+}|^2}{\lambda^4} + |\mathcal{A}_{D^0\to K^{*-}\pi^+}|^2 = \frac{|\mathcal{A}_{D^0\to K^{*-}K^+}|^2}{\lambda^2} + \frac{|\mathcal{A}_{D^0\to \rho^-\pi^+}|^2}{\lambda^2}$$

$$|\mathcal{A}_{D^0\to K^-\rho^+}|^2 + \frac{|\mathcal{A}_{D^0\to \pi^-K^{*+}}|^2}{\lambda^4} = \frac{|\mathcal{A}_{D^0\to \pi^-\rho^+}|^2}{\lambda^2} + \frac{|\mathcal{A}_{D^0\to K^-K^{*+}}|^2}{\lambda^2}$$

$\mathcal{O}(\epsilon^2)$ Rate Sum Rule Examples

Long Example:

$$\left[\frac{|\mathcal{A}_{D^0\to\phi K_L}|^2}{\lambda^2} - \frac{|\mathcal{A}_{D^0\to\phi K_S}|^2}{\lambda^2}\right] + \left[\frac{|\mathcal{A}_{D^0\to\omega K_L}|^2}{\lambda^2} - \frac{|\mathcal{A}_{D^0\to\omega K_S}|^2}{\lambda^2}\right]$$

$$+\left[\frac{|\mathcal{A}_{D^0\to K_L\rho_0}|^2}{\lambda^2} - \frac{|\mathcal{A}_{D^0\to K_S\rho_0}|^2}{\lambda^2}\right] + \frac{|\mathcal{A}_{D^0\to\eta K^{*0}}|^2}{\lambda^4} + \frac{|\mathcal{A}_{D^0\to\pi_0 K^{*0}}|^2}{\lambda^4} + \frac{|\mathcal{A}_{D^0\to K^{*0}\eta'}|^2}{\lambda^4}$$

$$+|\mathcal{A}_{D^0\to\eta\overline{K}^{*0}}|^2 + |\mathcal{A}_{D^0\to\pi_0\overline{K}^{*0}}|^2 + |\mathcal{A}_{D^0\to\overline{K}^{*0}\eta'}|^2$$

$$=$$

$$\frac{|\mathcal{A}_{D^0\to\eta\phi}|^2}{\lambda^2} + \frac{|\mathcal{A}_{D^0\to\eta\omega}|^2}{\lambda^2} + \frac{|\mathcal{A}_{D^0\to\phi\pi_0}|^2}{\lambda^2} + \frac{|\mathcal{A}_{D^0\to\omega\pi_0}|^2}{\lambda^2} + \frac{|\mathcal{A}_{D^0\to\eta\rho_0}|^2}{\lambda^2} + \frac{|\mathcal{A}_{D^0\to\pi_0\rho_0}|^2}{\lambda^2}$$

$$+\frac{|\mathcal{A}_{D^0\to\phi\eta'}|^2}{\lambda^2} + \frac{|\mathcal{A}_{D^0\to\omega\eta'}|^2}{\lambda^2} + \frac{|\mathcal{A}_{D^0\to\rho_0\eta'}|^2}{\lambda^2},$$

Many of these modes have not yet been measured.

U-spin

We saw amplitude and rate sum rules for U-spin. A special property of U-spin: normed amplitude sum rules

$$\frac{|\mathcal{A}_{D^0\to K^-K^+}|}{\lambda} + \frac{|\mathcal{A}_{D^0\to \pi^-\pi^+}|}{\lambda} = \frac{|\mathcal{A}_{D^0\to \pi^-K^+}|}{\lambda^2} + |\mathcal{A}_{D^0\to K^-\pi^+}|$$

$$|\mathcal{A}_{D^0\to K^-\rho^+}| + \frac{|\mathcal{A}_{D^0\to \pi^-K^{*+}}|}{\lambda^2} = \frac{|\mathcal{A}_{D^0\to \pi^-\rho^+}|}{\lambda} + \frac{|\mathcal{A}_{D^0\to K^-K^{*+}}|}{\lambda}$$

$$\frac{|\mathcal{A}_{D^0\to \rho^-K^+}|}{\lambda^2} + |\mathcal{A}_{D^0\to K^{*-}\pi^+}| = \frac{|\mathcal{A}_{D^0\to K^{*-}K^+}|}{\lambda} + \frac{|\mathcal{A}_{D^0\to \rho^-\pi^+}|}{\lambda}$$

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$\sim 4.0\% \pm 1.6\%$

$$|\mathcal{A}_{D^0\to K^-\rho^+}| + \frac{|\mathcal{A}_{D^0\to \pi^-K^{*+}}|}{\lambda^2} = \frac{|\mathcal{A}_{D^0\to \pi^-\rho^+}|}{\lambda} + \frac{|\mathcal{A}_{D^0\to K^-K^{*+}}|}{\lambda}$$

$\sim 6\% \pm 17\%$

$$\frac{|\mathcal{A}_{D^0\to \rho^-K^+}|}{\lambda^2} + |\mathcal{A}_{D^0\to K^{*-}\pi^+}| = \frac{|\mathcal{A}_{D^0\to K^{*-}K^+}|}{\lambda} + \frac{|\mathcal{A}_{D^0\to \rho^-\pi^+}|}{\lambda}$$

$\implies \mathrm{Br}(D^0 \to \rho^- K^+) \simeq (1.7 \pm 0.4) \times 10^{-4}$

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Predictions

- U-spin sum rules admit the $\Delta U = 0$ rule

$$\begin{gathered}
\mathcal{A}_{D^0\to K^\pm K^{*\mp}} \simeq \lambda[T^\pm - P_\mathrm{b}^\pm] - \lambda^5 e^{i(\delta^\pm-\gamma)} P^\pm \ , \\
\mathcal{A}_{D^0\to \pi^\pm \rho^\mp} \simeq -\lambda[T^\pm + P_\mathrm{b}^\pm] - \lambda^5 e^{i(\delta^\pm-\gamma)} P^\pm \ , \\
\mathcal{A}_{D^0\to K^{*+}\pi^-} \simeq \lambda^2 T^+ \ , \quad \mathcal{A}_{D^0\to K^{*-}\pi^+} \simeq T^- \ , \\
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\end{aligned}$$

- This picture implies

$$\frac{\mathcal{A}_{\mathrm{CP}}(K^+K^{*-})}{\mathcal{A}_{\mathrm{CP}}(\pi^+\rho^-)} \simeq -1.59 \pm 0.10\ ,\quad \frac{\mathcal{A}_{\mathrm{CP}}(K^-K^{*+})}{\mathcal{A}_{\mathrm{CP}}(\pi^-\rho^+)} \simeq -1.33 \pm 0.05\ ,$$

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- And up to $\mathcal{O}(1)$

$$\Delta\mathcal{A}_{\mathrm{CP}}^{\pm} \sim -\Delta\mathcal{A}_{\mathrm{CP}}$$

Summary

- Existing data can be accomodated by $\cancel{\mathrm{SU}(3)}$ patterns combined with non-perturbative effects.
- These same patterns of $\cancel{\mathrm{SU}(3)}$ imply amplitude and rate sum rules: we now know what they all are for PP and PV final states.
- Isospin sum rules are extremely sensitive to new $\cancel{\mathrm{SU}(3)}$ sources; U-spin sum rules imply predictions for branching ratios and $\Delta\mathcal{A}_{\mathrm{CP}}$ in the PV system.

Thank you!